TEMPERATURE INVERSION ON THE SURFACE OF EXTERNALLY HEATED OPTICALLY THICK MULTIGRAIN DUST CLOUDS

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ABSTRACT

It was recently discovered that the temperature in the surface layer of externally heated optically thick gray dust clouds increases with the optical depth for some distance from the surface, as opposed to the normal decrease in temperature with distance in the rest of the cloud. This temperature inversion is a result of efficient absorption of diffuse flux from the cloud interior by the surface dust exposed to external radiation. Grains of size 1 μm or bigger experience this effect when the external flux is of stellar spectrum. We explore what happens to the effect when dust is a mixture of grain sizes (multigrain). Two possible boundary conditions are considered: (1) a constant external flux without constrains on the dust temperature, and (2) the maximum dust temperature set to the sublimation temperature. We find that the first condition allows small grains to completely suppress the temperature inversion of big grains if the overall opacity is dominated by small grains. The second condition enables big grains to maintain the inversion even when they are a minor contributor to the opacity. In reality, the choice of boundary condition depends on the dust dynamics. When applied to the physics of protoplanetary disks, the temperature inversion leads to a previously unrecognized disk structure in which optically thin dust can exist inside the dust destruction radius of an optically thick disk. We conclude that the transition between the dusty disk and the gaseous inner clearing is not a sharp edge, but rather a large optically thin region.

Subject headings: accretion, accretion disks — circumstellar matter — dust, extinction — stars: pre–main-sequence

Online material: color figure

1. INTRODUCTION

Dust is one of the principal components of interstellar and circumstellar matter. It serves as a very efficient absorber of starlight, which is dominated by visual and ultraviolet photons, and reemiter of this absorbed energy into the infrared (IR), thereby modifying the entire spectral energy distribution. Hence, radiative transfer in dusty environments is an inherent part of the study of star formation, protoplanetary disk evolution, dusty winds from AGB stars, circumnuclear environment of active galactic nuclei (AGNs), etc.

In general, radiative transfer is extremely complicated because the dust in these environments always comes as a mixture of various grain sizes (multigrain[1]), shapes and chemical compositions. It is therefore common to employ certain approximations that make the problem computationally manageable. For example, dust grains are usually approximated with spheres of similar chemical composition, so that the Mie theory can be easily used for calculating dust cross sections. Another commonly used approximation is to replace a mixture of dust grain sizes by an equivalent (i.e., synthetic or average) single grain size. Numerical calculations have shown that this approximation does not produce a significant change in the spectral energy distribution (Efstathiou & Rowan-Robinson 1994; Wolf 2003; Carciofi et al. 2004). It is assumed that this is particularly appropriate for optically thick dust clouds.

It has been discovered recently that externally heated optically thick clouds made of large ($\gtrsim$1 μm) single-size dust grains produce the effect of *temperature inversion*, where the maximum tem-

perature is within the dust cloud,[2] at the visual optical depth of $\tau_V \sim 1$, instead of on the very surface exposed to the stellar heating (Isella & Natta 2005; Vinković et al. 2006). The cause of this inversion within the surface layer is its ability to efficiently absorb the diffuse IR radiation originating from the cloud's interior—a process similar to the "greenhouse effect."

It is not known, however, whether multigrain dust would produce the same temperature inversion effect. Here we employ analytical multigrain radiative transfer to study the surface of optically thick clouds. Two possible types of boundary conditions are explored: (1) a constant external flux heating the cloud, with no limits on the dust temperature, and (2) a fixed maximum dust temperature, corresponding to dust sublimation. In § 2 we describe the analytic method and apply it to single-size dust. In the next section, § 3, we apply the method to multigrain dust. After that, in § 4 we explore the possibility of surface thermal cooling in a transverse direction. Some aspects of our result are discussed in § 5, with our conclusions in § 6.

2. OPTICALLY THICK SINGLE-SIZE-GRAIN DUST CLOUD

In order to understand effects of multigrain dust on the temperature structure of optically thick clouds, we first have to take a look at the single-size dust grain clouds. Here we reiterate analytic techniques described in the literature (Isella & Natta 2005; Vinković et al. 2006) and then expand our approach to multigrain dust in the following sections.

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[1] Typically, the term *multigrain* also includes all other dust grain properties, such as the grain shape and chemistry, but for simplicity we use this term only to describe grain size effects.

[2] Another type of temperature inversion has been recognized in protoplanetary disks, where additional viscous heating can increase the disk interior temperature (Calvet et al. 1991; Malbet & Bertout 1991). In contrast, the temperature inversion discussed here is a pure radiative transfer effect and does not require any additional assumption (such as disk viscosity) to operate.
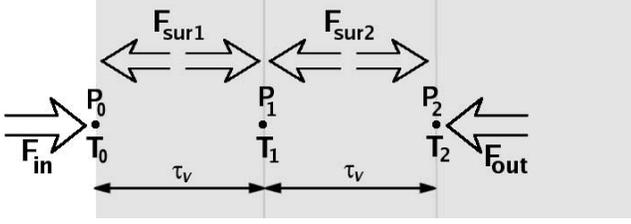
FIG. 1.—Sketch of the surface of an optically thick single-size-grain dust cloud illuminated from outside. See § 2 for details.

Consider dust grains at three different locations (Fig. 1): on the very surface (point P_0), at optical depth τ_V (point P_1) and at optical depth $2\tau_V$ (point P_2). The optical depth τ_V is defined at the peak wavelength of the bolometric temperature of external flux F_{in} illuminating the cloud. We are interested in cases where dust can reach sublimation temperatures (between $\sim$1000 and 2000 K for interstellar dust) and where the external flux source is a starlike object whose emission peaks at submicron wavelengths. For the purpose of this paper we use the optical depth τ_V in visual, but one can adjust it to the required wavelength of interest.

The surface layer between points P_0 and P_1 emits the thermal flux F_{sur1}. We assume that it emits equally on both sides (toward the left and right in Fig. 1). This approximation is valid if the layer is optically thin at the peak wavelength of its IR emission. If we define $q = \sigma_V/\sigma_{IR}$, where σ_V and σ_{IR} are dust absorption cross sections in visual and IR, then the IR optical depth requirement is

$$\tau_{IR} = \frac{\tau_V}{q} \lesssim 1. \tag{1}$$

The same requirement holds for the second surface layer between points P_1 and P_2 and its thermal flux F_{sur2}. Each of these two layers attenuates the external flux by $e^{-\tau_V}$ and the thermal fluxes by $e^{-\tau_{IR}} = e^{-\tau_V/q}$.

Since the cloud is optically thick to its own radiation, no net flux can go through the cloud. The total flux coming from the left at any point in the cloud is equal to the total flux coming from the right. This yields flux balance equations at points P_0, P_1, and P_2:

$$F_{in} = F_{sur1} + F_{sur2}e^{-\tau_V/q} + F_{out}e^{-2\tau_V/q}, \tag{2}$$

$$F_{in}e^{-\tau_V} + F_{sur1} = F_{sur2} + F_{out}e^{-\tau_V/q}, \tag{3}$$

$$F_{in}e^{-2\tau_V} + F_{sur1}e^{-\tau_V/q} + F_{sur2} = F_{out}, \tag{4}$$

where F_{out} is the thermal flux coming out of the cloud interior at P_2.

We want the external flux F_{in} at point P_2 to be attenuated enough to make diffuse flux the dominant source of heating. This requirement means that $2\tau_V \gtrsim 1$, which in combination with equation (1) gives the allowed range for τ_V,

$$0.5 \lesssim \tau_V \lesssim q. \tag{5}$$

From equations (2)–(4) we further get

$$F_{sur2} = F_{in}\frac{1 + e^{-\tau_V}}{1 + e^{-\tau_V/q}} - F_{out}e^{-\tau_V/q}, \tag{6}$$

$$F_{sur1} = F_{sur2} - F_{in}e^{-\tau_V}\frac{1 + e^{-\tau_V}}{1 + e^{-\tau_V/q}}. \tag{7}$$

A dust grain at P_0 absorbs $\sim\sigma_V F_{in}$ of the external directional flux. It also absorbs $\sim 2\sigma_{IR}F_{IR}$ of any infrared diffuse flux F_{IR}, where the factor 2 accounts for absorption from 2π sr. The grain emits at its temperature T_0 into 4π sr, so that the energy balance is

$$\sigma_V F_{in} + 2\sigma_{IR}(F_{sur1} + F_{sur2}e^{-\tau_V/q} + F_{out}e^{-2\tau_V/q}) = 4\sigma_{IR}\sigma_{SB}T_0^4, \tag{8}$$

where σ_{SB} is the Stefan-Boltzmann constant. Using the flux balance in equation (2) and $q = \sigma_V/\sigma_{IR}$, we get

$$F_{in} = \frac{4\sigma_{SB}}{q+2}T_0^4. \tag{9}$$

Similarly, we can write the energy balance for a dust grain at P_1 and P_2,

$$qF_{in}e^{-\tau_V} + 2F_{sur1} + 2F_{sur2} + 2F_{out}e^{-\tau_V/q} = 4\sigma_{SB}T_1^4, \tag{10}$$

$$qF_{in}e^{-2\tau_V} + 2F_{sur1}e^{-\tau_V/q} + 2F_{sur2} + 2F_{out} = 4\sigma_{SB}T_2^4. \tag{11}$$

We approximate the interior flux as

$$F_{out} \sim \sigma_{SB}T_2^4. \tag{12}$$

This is a good approximation of the interior for gray dust and an overestimate for nongray dust, where temperature decreases with optical depth. Now we can continue deriving temperatures in two possible ways.

Method 1.—Combining equations (6), (7), (9), (10), (11), and (12) yields

$$T_1^4 = \frac{A+B}{q+2}T_0^4 \tag{13}$$

$$T_2^4 = \frac{2B}{q+2}T_0^4 \tag{14}$$

$$A = qe^{-\tau_V} + 2(1 - e^{-\tau_V})\frac{1 + e^{-\tau_V}}{1 + e^{-\tau_V/q}} \tag{15}$$

$$B = qe^{-2\tau_V} + 2\left(1 - e^{-\tau_V - \tau_V/q}\right)\frac{1 + e^{-\tau_V}}{1 + e^{-\tau_V/q}}. \tag{16}$$

The upper panel of Figure 2 shows this temperature profile for $\tau_V = 1$. The effect of temperature inversion $T_2 > T_0$ is present when $q < q_{lim} \sim 1.4$. Only grains larger than about 1 μm can have such a small value of q, which makes them almost "gray" in the near-IR. This inversion does not appear in nongray dust ($q > q_{lim}$), where the temperature decreases monotonically with distance from the cloud surface.

Method 2.—We can use equations (2)–(4) to express F_{out} as a function of F_{in} and then use equations (9) and (12) to obtain

$$T_2^4 = \frac{4}{q+2}\frac{1 + e^{-\tau_V}\left(1 + e^{-\tau_V} - e^{-\tau_V/q}\right)}{1 + e^{-\tau_V/q}}T_0^4. \tag{17}$$

Similarly, we can express F_{sur1} and F_{sur2} as a function of F_{in} and then use equation (10) to obtain

$$T_1^4 = \frac{1}{q+2}\left(qe^{-\tau_V} + 2\frac{2 + e^{-\tau_V} - e^{-\tau_V - \tau_V/q}}{1 + e^{-\tau_V/q}}\right)T_0^4. \tag{18}$$
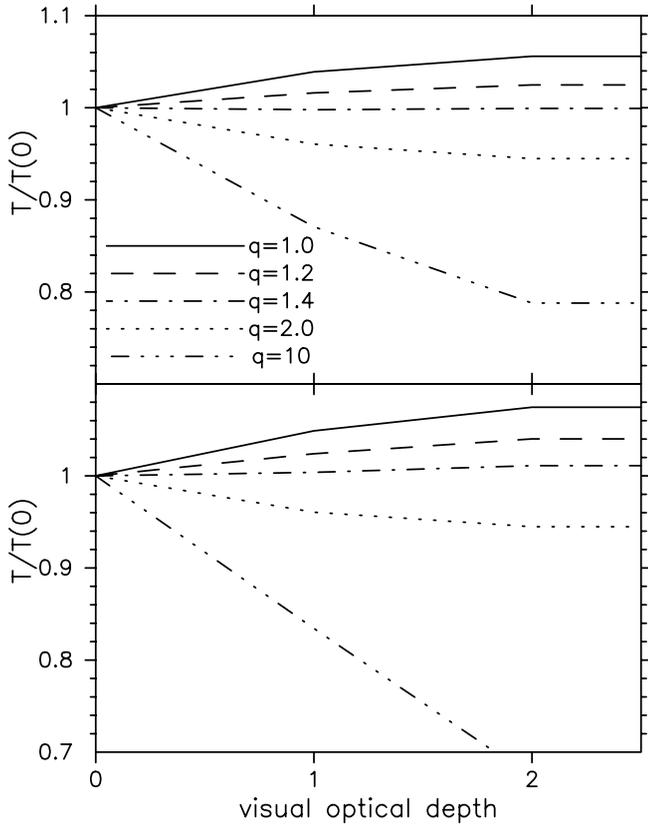
FIG. 2.—Surface temperature of an optically thick single-size-grain dust cloud for various visual-to-infrared opacity ratios q. The upper panel shows the solution based on eqs. (13) and (14) (method 1). The lower panel is based on eqs. (17) and (18) (method 2). Optical depth step is $\tau_V = 1$. Both solutions show temperature inversion (increase of temperature with optical depth) for small q.

The lower panel of Figure 2 shows this result for $\tau_V = 1$. It differs slightly from previous solution in method 1, but gives qualitatively the same result. The temperature inversion now exists for $q < q_{\mathrm{lim}} \sim 1.5$. Henceforth, we use method 1 for further analysis.

3. OPTICALLY THICK MULTIGRAIN CLOUD

3.1. *Constant External Flux as the Boundary Condition*

We consider now an externally heated optically thick dust cloud made of N grain sizes, under the condition of constant external flux and no limits on the dust temperature. A dust grain of the ith size at a point of τ_V optical distance from the cloud surface is heated by the attenuated stellar flux $F_{\mathrm{in}}e^{-\tau_V}$, by the local diffuse flux F_{sur} coming from the direction of the surface, and by the local diffuse flux F_{out} coming out of the cloud interior. The thermodynamic equilibrium of the dust grain gives

$$\sigma_{V,i}F_{\mathrm{in}}e^{-\tau_V} + 2\sigma_{\mathrm{IR},i}(F_{\mathrm{sur}} + F_{\mathrm{out}}) = 4\sigma_{\mathrm{IR},i}\sigma_{\mathrm{SB}}T_i^4(\tau_V). \quad (19)$$

The factors 2 and 4 account for absorption from 2π sr and emission into 4π sr.

On the very surface of the cloud $\tau_V = 0$ and $F_{\mathrm{sur}} = 0$, and therefore $F_{\mathrm{in}} = F_{\mathrm{out}}$, which yields

$$T_i^4(0) = (q_i + 2)\frac{F_{\mathrm{in}}}{4\sigma_{\mathrm{SB}}}, \quad (20)$$

where $q_i = \sigma_{V,i}/\sigma_{\mathrm{IR},i}$. This shows that *dust grains of different sizes have different temperatures on the surface of an optically thick cloud*. The lowest temperature among the grain sizes is acquired by the largest grains, because they have the smallest q_i.

On the other hand, the contribution of the external flux is negligible; $F_{\mathrm{in}}e^{-\tau_V} \to 0$ when $\tau_V \gg 1$, and therefore $F_{\mathrm{sur}} = F_{\mathrm{out}}$, which yields the same temperature for all grain sizes,

$$T_i^4(\tau_V \gg 1) = F_{\mathrm{out}}/\sigma_{\mathrm{SB}}. \quad (21)$$

A general relationship between grain temperatures is derived by subtracting equation (19) for a grain i from the same equation for a grain j and then using equation (20) to obtain

$$T_j^4(\tau_V) = T_i^4(\tau_V) + \frac{q_j - q_i}{q_i + 2}e^{-\tau_V}T_i^4(0). \quad (22)$$

Since q scales inversely with the grain size, equation (22) shows that *smaller grains always have a higher temperature than bigger grains at any point in the cloud*, with the limit $T_j \sim T_i$ when $e^{-\tau_V} \ll 1$.

Optical depth is now a cumulative contribution of all grain sizes in the mix;

$$\tau_\lambda \propto \sum_{i=1}^{N} n_i\sigma_{\lambda,i}, \quad (23)$$

where n_i is the number density of the ith grain size. If we scale optical depth relative to τ_V, then

$$\tau_{\mathrm{IR}} = \tau_V\frac{\sum_{i=1}^{N} n_i\sigma_{\mathrm{IR},i}}{\sum_{i=1}^{N} n_i\sigma_{V,i}} = \tau_V\sum_{i=1}^{N}\frac{\Upsilon_{V,i}}{q_i}, \quad (24)$$

where $\Upsilon_{\lambda,i}$ is the relative contribution of the ith grain to the dust opacity at wavelength λ,

$$\Upsilon_{\lambda,i} = \frac{n_i\sigma_{\lambda,i}}{\sum_{j=1}^{N} n_j\sigma_{\lambda,j}}. \quad (25)$$

Consider now a model similar to Figure 1, except that the dust is multigrain. For simplicity and clarity of the following analysis, we work with only two grain sizes: a "big" grain α with $q_\alpha \sim 1$ and a "small" grain β with $q_\beta > q_\alpha$. The infrared optical depth step is

$$\tau_{\mathrm{IR}} = \tau_V\left(\frac{\Upsilon_{V,\alpha}}{q_\alpha} + \frac{\Upsilon_{V,\beta}}{q_\beta}\right). \quad (26)$$

According to equation (22), the temperature of small grains at optical depth $k\tau_V$ (point P_k) is

$$T_{k,\beta}^4 = T_{k,\alpha}^4 + \frac{q_\beta - q_\alpha}{q_\alpha + 2}e^{-k\tau_V}T_{0,\alpha}^4. \quad (27)$$

The temperature of big grains at point P_0 is (eq. [20])

$$T_{0,\alpha}^4 = (q_\alpha + 2)\frac{F_{\mathrm{in}}}{4\sigma_{\mathrm{SB}}}, \quad (28)$$

while for the other temperatures we need the flux balance at points P_0, P_1, and P_2. Since the balance is the same as in equations (2)–(4),
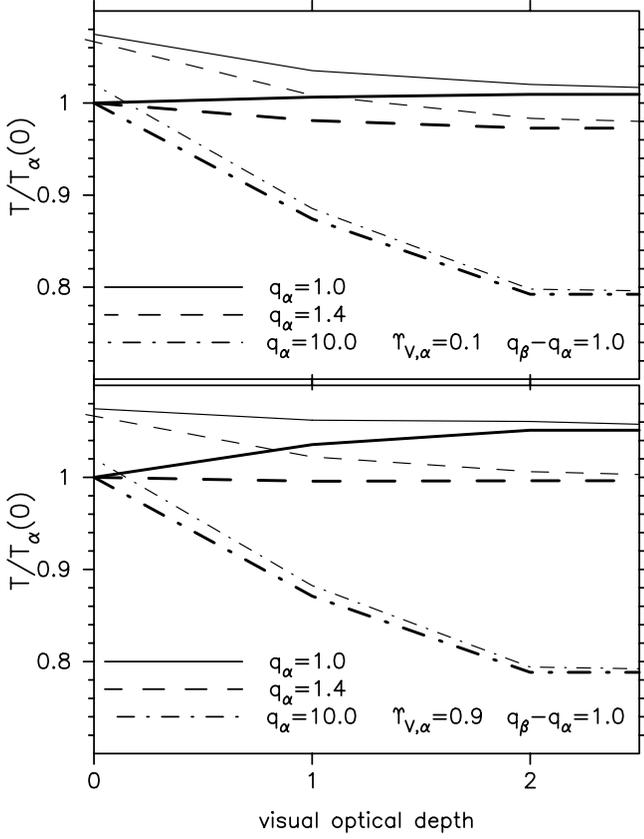
Fig. 3.—Temperature of two types of grains in the surface of an optically thick multigrain dust cloud: larger grains α (*thick lines*; eqs. [36] and [37]) and smaller grains β (*thin lines*; eq. [27]). Line styles correspond to different ratios q_α of visual to infrared opacity of big grains. The same ratio for small grains is fixed to $q_\beta = q_\alpha + 1$. Panels show results for two relative contributions of big grains to the visual opacity: $\Upsilon_{V\alpha} = 0.1$ in the upper panel and $\Upsilon_{V\alpha} = 0.9$ in the lower panel. Optical depth step is $\tau_V = 1$. Note how the temperature inversion in big grains (small q_α) disappears when small grains dominate the overall opacity (*upper panel*). The inversion exists only if bigger grains dominate (*lower panel*).

Fig. 4.—Same as Fig. 3, but for $q_\beta = q_\alpha + 10$.

except that the IR step τ_V/q is replaced by τ_{IR}, we use the procedure described in § 2 and derive

$$T_{1,\alpha}^4 = \frac{A_\alpha}{q_\alpha + 2} T_{0,\alpha}^4 + \frac{F_{out}}{2\sigma_{SB}}, \qquad (29)$$

$$T_{2,\alpha}^4 = \frac{B_\alpha}{q_\alpha + 2} T_{0,\alpha}^4 + \frac{F_{out}}{2\sigma_{SB}}, \qquad (30)$$

$$A_\alpha = q_\alpha e^{-\tau_V} + 2(1 + e^{-\tau_V}) \frac{1 + e^{-\tau_V}}{1 + e^{-\tau_{IR}}}, \qquad (31)$$

$$B_\alpha = q_\alpha e^{-2\tau_V} + 2(1 + e^{-\tau_V - \tau_{IR}}) \frac{1 + e^{-\tau_V}}{1 + e^{-\tau_{IR}}}. \qquad (32)$$

The interior flux F_{out} is now a cumulative contribution of all grain sizes according to their relative contribution to the dust opacity. In our two-size example,

$$\frac{F_{out}}{\sigma_{SB}} = \Upsilon_{IR,\alpha} T_{2,\alpha}^4 + \Upsilon_{IR,\beta} T_{2,\beta}^4. \qquad (33)$$

Combined with equation (27) gives

$$\frac{F_{out}}{\sigma_{SB}} = T_{2,\alpha}^4 + \frac{C_\alpha}{q_\alpha + 2} T_{0,\alpha}^4, \qquad (34)$$

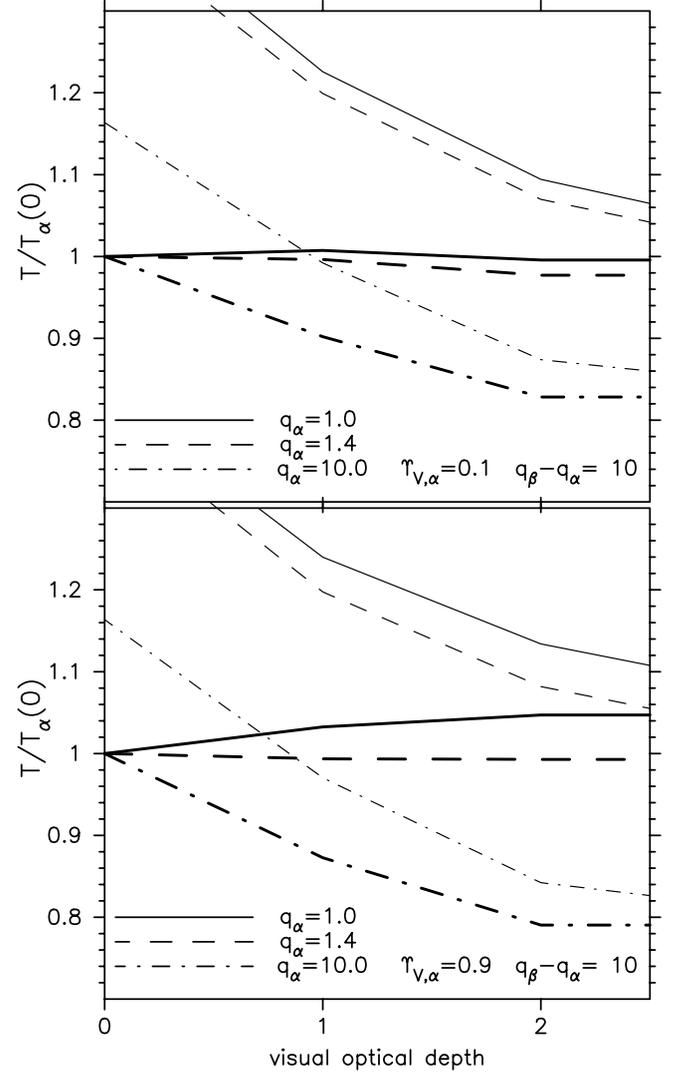$$C_\alpha = \Upsilon_{IR,\beta}(q_\beta - q_\alpha) e^{-2\tau_V}. \qquad (35)$$

Putting together equations (29), (30), and (34) yields the solution

$$T_{1,\alpha}^4 = \frac{A_\alpha + B_\alpha + C_\alpha}{q_\alpha + 2} T_{0,\alpha}^4, \qquad (36)$$

$$T_{2,\alpha}^4 = \frac{2B_\alpha + C_\alpha}{q_\alpha + 2} T_{0,\alpha}^4. \qquad (37)$$

The resulting temperature is plotted in Figures 3 and 4. The upper panel in each figure shows the result when small grains dominate the opacity ($\Upsilon_{V,\alpha} = 0.1$). The temperature inversion in big grains is suppressed because the local diffuse flux is dictated by small grains. If big grains dominate the opacity (*lower panels*; $\Upsilon_{V,\alpha} = 0.9$), then the temperature inversion is preserved.

3.2. *Sublimation Temperature as the Boundary Condition*

Now we consider a dust cloud hot enough on its illuminated surface to sublimate dust grains warmer than the sublimation temperature T_{sub}. The flux entering the cloud is adjustable to accommodate any temperature boundary condition. From equation (20) we see that small grains are the first to be removed from the immediate surface. If the external flux is high enough, then the immediate surface is populated only by $q_i \sim 1$ grains ("big
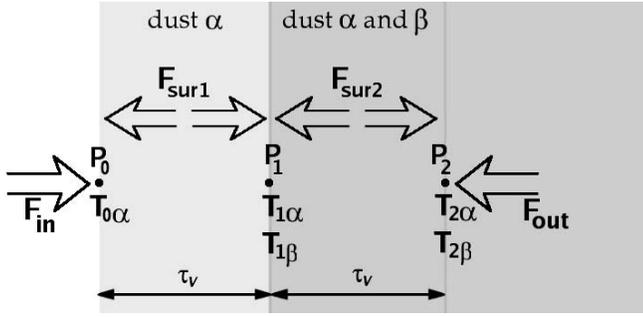
Fig. 5.—Sketch of the surface of an optically thick multigrain dust cloud illuminated from outside. See § 3.2 for details.

grains"). All other grains ("small grains") would survive somewhere within the cloud, at a distance where the local flux is reddened enough by big grains to be absorbed less efficiently.

We again apply our two-size dust model, except that the surface layer of visual optical depth τ_V is occupied only by big grains (see Fig. 5). It is too hot for small grains to survive within this layer. Both grains exist at optical distances larger than τ_V from the surface. Following the same procedure as in § 2, we write the flux balance at points P_0, P_1, and P_2 (see Fig. 5)

$$F_{\rm in} = F_{\rm sur1} + F_{\rm sur2}e^{-\tau_V/q_\alpha} + F_{\rm out}e^{-\tau_V/q_\alpha - \tau_{\rm IR}} \quad (38)$$

$$F_{\rm in}e^{-\tau_V} + F_{\rm sur1} = F_{\rm sur2} + F_{\rm out}e^{-\tau_{\rm IR}} \quad (39)$$

$$F_{\rm in}e^{-2\tau_V} + F_{\rm sur1}e^{-\tau_{\rm IR}} + F_{\rm sur2} = F_{\rm out}, \quad (40)$$
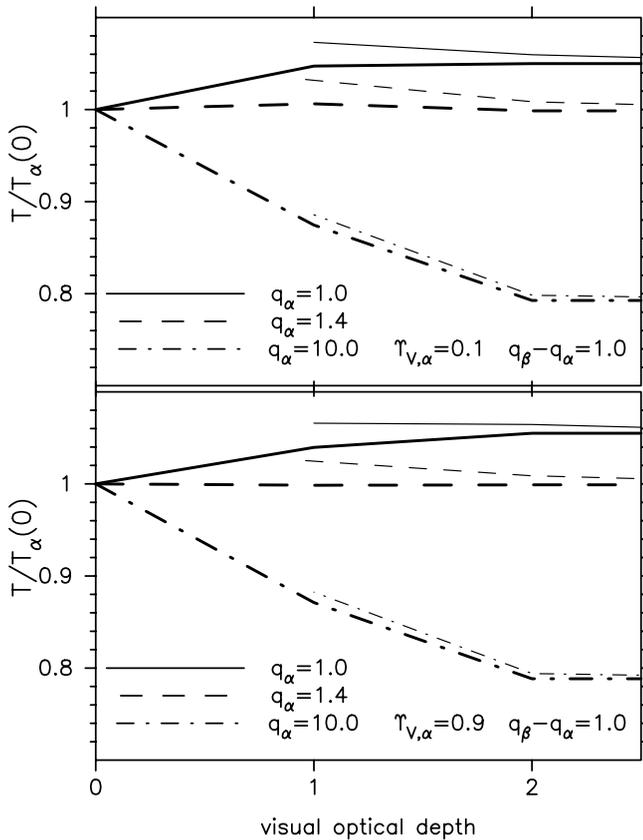

Fig. 6.—Same as Fig. 3, except that small grains are removed from the immediate surface by sublimation (Fig. 5). The temperature of larger grains is calculated from eqs. (43) and (44). Despite being mixed with smaller grains, larger grains show temperature inversion even when smaller grains dominate the opacity.
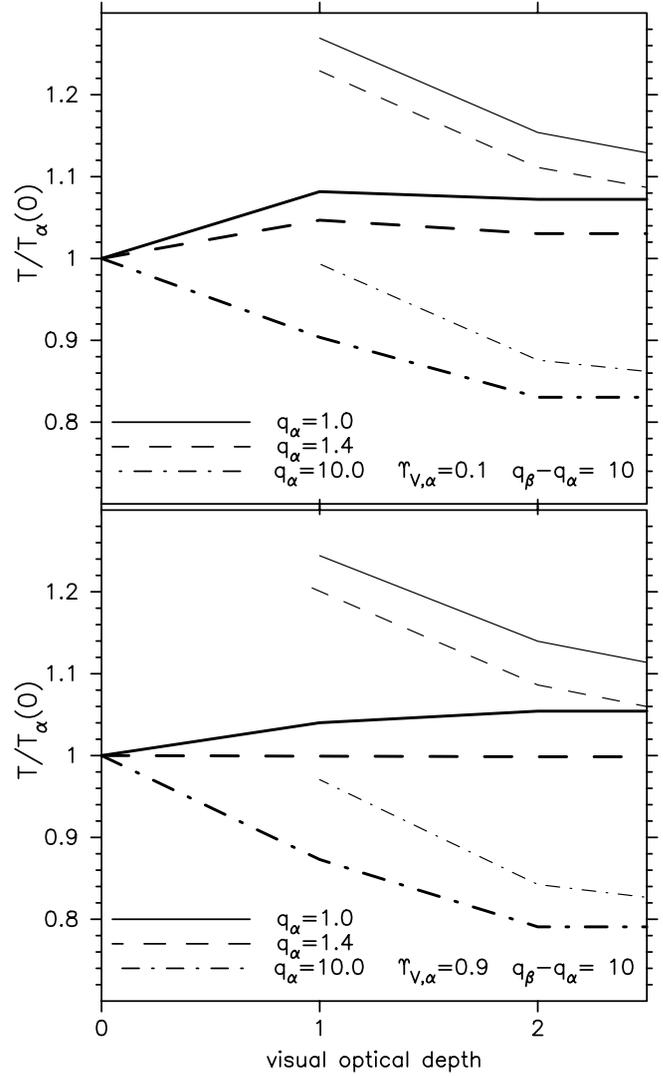

Fig. 7.—Same as Fig. 6, but for $q_\beta = q_\alpha + 10$.

where the IR optical depth between points P_1 and P_2 is given in equation (26).

Small grains do not exist now at point P_0, but their temperature at other points is still described by equation (27). Deriving big grain temperatures at P_1 and P_2 is now a straightforward procedure already described in § 2 and § 3.1:

$$A'_\alpha = q_\alpha e^{-\tau_V} + 2\frac{1+e^{-\tau_V}}{1+e^{-\tau_V/q_\alpha}} - 2e^{-\tau_V}\frac{1+e^{-\tau_V}}{1+e^{-\tau_{\rm IR}}} \quad (41)$$

$$B'_\alpha = q_\alpha e^{-2\tau_V} + 2\frac{1+e^{-\tau_V}}{1+e^{-\tau_V/q_\alpha}} - 2e^{-\tau_V - \tau_{\rm IR}}\frac{1+e^{-\tau_V}}{1+e^{-\tau_{\rm IR}}} \quad (42)$$

$$T_{1,\alpha}^4 = \frac{A'_\alpha + B'_\alpha + C_\alpha}{q_\alpha + 2}T_{0,\alpha}^4 \quad (43)$$

$$T_{2,\alpha}^4 = \frac{2B'_\alpha + C_\alpha}{q_\alpha + 2}T_{0,\alpha}^4. \quad (44)$$

The resulting temperature is plotted in Figures 6 and 7. Two important results are deduced for big grains from this solution: (1) big grains maintain the temperature inversion and (2) if small grains set the maximum temperature limit then big grains, which are always colder than small grains (see eq. [22]), cannot reach the sublimation temperature.
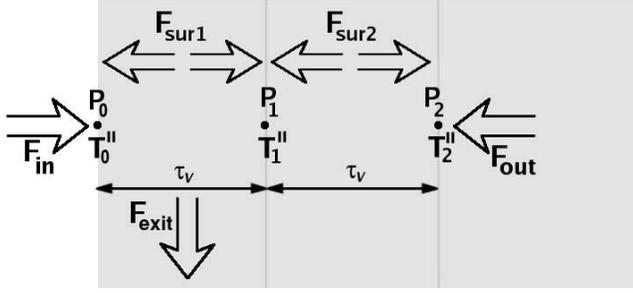
Fig. 8.—Sketch of the surface of an optically thick dust cloud illuminated from outside and cooled through the surface layer in the transverse direction. See § 4 for details.
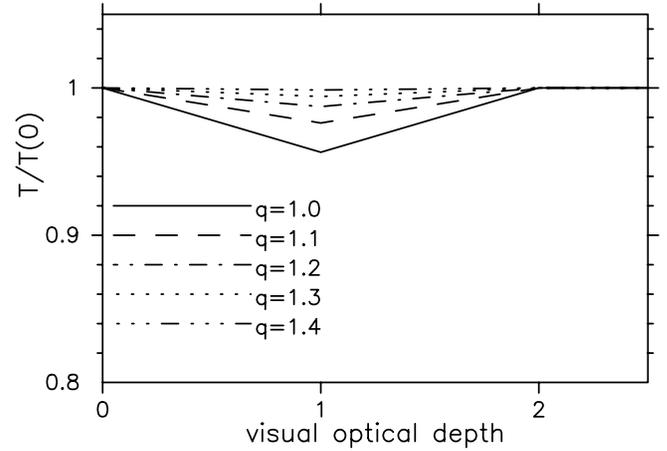

Fig. 9.—Surface temperature of an optically thick single-size-grain dust cloud cooled transversely through the surface layer (eqs. [47] and [48]). The amount of transverse cooling is adjusted to the maximum temperature $T(0)$ at the very surface (eq. [49]). Line styles correspond to different ratios q of visual to infrared opacity. Optical depth step is $\tau_V = 1$. Increase in $T(0)$, combined with the temperature inversion (see also Fig. 2), creates a local temperature minimum within the surface layer.

However, since we do not set limits on the external flux, the solution that allows the maximum possible external flux is the one that also maximizes $F_{\rm out}$. From equation (33) we see that the maximum $F_{\rm out} = \sigma_{\rm SB} T_{\rm sub}$ is achieved when $T_{2,\alpha} = T_{2,\beta} = T_{\rm sub}$. Such a solution is not possible unless $\Upsilon_{{\rm IR},\beta} = 0$. Therefore, dust temperatures are maximized when all small grains sublimate away from the surface region and exist only at optical depths of $e^{-\tau_V} \ll 1$. In other words, the external flux is maximized when the cloud surface "belongs" exclusively to big grains only.

4. TRANSVERSE COOLING OF THE CLOUD SURFACE

Solutions presented so far assume no time variability in any of the model parameters. The external flux is adjusted by hand to the value that maximizes the external flux and keeps dust temperatures below sublimation. In reality, however, this is a dynamical process in which the equilibrium is established by dust moving around and sublimating whenever its temperature exceeds the sublimation point.

The existence of temperature inversion is difficult to understand under such dynamical conditions. Since the very surface of the cloud is *below* the sublimation point, its dust can survive closer to the external energy source than the dust within the cloud. Therefore, the distance of the cloud from the source is not defined by the very surface, but rather by the dust at the peak temperature *within* the cloud. Imagine now that the whole cloud is moving closer to the source. From the cloud point of reference, this dynamical process is equivalent to increasing the external flux. According to solutions in § 2 and § 3, the peak temperature within the cloud will exceed sublimation at a certain distance from the source, and the dust will start to sublimate. This point is the distance at which the dust cloud seemingly stops. However, there is nothing to stop the very surface layer of the cloud from moving even closer than the rest of the cloud, because its temperature is below sublimation.

Note that we cannot resolve this issue in the approximation of an infinite dusty slab, because the transverse optical depth (parallel to the slab surface) is always infinite. No flux can escape the slab in the transverse direction. Hence, the radiative transfer in a slab does not depend on spatial scale; only the optical depth matters. The spatial extension of the surface layer is irrelevant, and solutions from § 2 and § 3 are applicable irrespective of the dust dynamics on the spatial scale.

In reality, however, the cloud is finite and dust sublimation can eventually make the cloud optically thin in the transverse direction. This optical depth gap enables thermal radiation to flow out in the transverse direction and thus provides a channel for thermal cooling. Under such conditions the surface layer can move closer and closer to the energy source by simultaneously expand-

ing the size of the transverse optical depth gap, which then increases the amount of escaping thermal flux. Since the very surface dust is getting closer to the energy source, its temperature increases and eventually it reaches the dust sublimation. At that moment the radiative and dynamical equilibria are established and the whole dust cloud seems to stop.

The resulting dust cloud has a large surface zone where the thermal flux can transversely escape the cloud. Here we make an attempt to describe this on a quantitative level through a model shown in Figure 8. It is similar to the infinite slab model described in § 2 except that now we introduce the flux $F_{\rm exit}$ exiting the cloud transversely through the surface layer.

The flux-balance equations (3) and (4) remain unchanged, while equation (2) is changed to

$$F_{\rm in} = F_{\rm exit} + F_{\rm sur1} + F_{\rm sur2}e^{-\tau_V/q} + F_{\rm out}e^{-2\tau_V/q}. \quad (45)$$

By following the same procedure as in § 2, we obtain temperatures T_k'' at points P_k,

$$F_{\rm in} = \frac{4\sigma_{\rm SB}}{q+2}T_0''^4 + \frac{2}{q+2}F_{\rm exit}, \quad (46)$$

$$T_1''^4 = \frac{A+B}{q+2}T_0''^4 - \left(\frac{1}{1+e^{-\tau_V/q}} - \frac{A+B}{2(q+2)}\right)F_{\rm exit}, \quad (47)$$

$$T_2''^4 = \frac{2B}{q+2}T_0''^4 - \left(\frac{1}{1+e^{-\tau_V/q}} - \frac{B}{q+2}\right)F_{\rm exit}, \quad (48)$$

where A and B are given in equations (15) and (16). The values in parentheses are always positive.

We compare this with the original solution in equations (9), (13), and (14). Note that keeping the interior temperature the same as before ($T_2'' = T_2$) requires a larger surface temperature ($T_0'' > T_0$). If the surface dust of T_0'' is dynamically driven toward the external energy source, as we discussed above, then it will eventually reach the sublimation temperature $T_{\rm sub}$. This dynamical process is accompanied by dust sublimation in the cloud interior. Sublimation creates a transverse optical depth gap and maintains

$T_2'' = T_{sub}$. The established equilibrium condition $T_0'' = T_2'' = T_{sub}$ yields

$$F_{exit} = \frac{2B - q - 2}{(q+2)/(1 + e^{-\tau_V/q}) - B} \, T_{sub}^4. \quad (49)$$

This flux is positive for values of q that produce temperature inversion ($q < q_{lim}$). It originates from the diffuse radiation trapped within the cloud. Figure 9 shows temperature profiles for $q < q_{lim}$ based on F_{exit} from equation (49). The surface layer now has a local temperature minimum at T_1''.

5. DISCUSSION

According to equation (21), the temperature of dust optically deep inside an optically thick cloud depends solely on the local diffuse flux. Since this can give a misleading impression that temperature effects on the cloud surface have no influence on the cloud interior, it is worth explaining why the surface solution is so important for the overall solution.

At optical depths $\tau_V \gtrsim 2$, the flux is exclusively diffuse and the dust temperature is dictated by equation (21). Note, however, that even though we set the limit of zero net flux flowing through the cloud, there is no limit on the absolute value of the local diffuse flux. Arbitrarily large but equal diffuse fluxes can flow in opposite directions, producing the net zero flux. It is the radiative transfer in the surface layer that sets the absolute scale. The cloud surface of $\tau_V \lesssim 2$ converts the directional external flux of stellar spectral shape into the diffuse thermal flux. Hence, this conversion determines the value of diffuse flux at $\tau_V \sim 2$, which then propagates into the cloud interior of $\tau_V \gtrsim 2$.

A more rigorous analytic approach shows that the net flux is not exactly zero. A very small flux, in comparison with the external flux, goes through the cloud and creates a temperature gradient. This yields the well-known gray opacity solution (Mihalas 1978), where $T^4(\tau) = a\tau + b$. While the constant a depends only on the net flux ($a = 0$ in the case of zero net flux), the constant b is determined by the boundary condition on the cloud surface. Hence, this becomes a reiteration of the role of the surface layer described above.

The possibility of the temperature-inversion effect described in this paper was discussed previously by Wolf (2003). He noticed in his numerical calculations that larger grains can have a higher temperature than smaller grains on the surface of circumstellar dusty disks. He correctly attributed this effect to the more efficient heating of large grains by the IR radiation from the disk interior, but did not analyze it further. This is the same effect noticed by Dullemond (2002) in his numerical models of circumstellar disks with gray dust opacities. However, it was not until Isella & Natta (2005) and Vinković et al. (2006) described this effect analytically in more detail that its importance to the overall disk structure was finally recognized.

The main driver for the detailed description of the temperature inversion effect came from the need for a better understanding of inner regions of dusty disks around young pre–main-sequence stars. The advancements of near-infrared interferometry enabled direct imaging of these inner disk regions and resulted in the discovery of inner disk holes produced by dust sublimation (Monnier & Millan-Gabet 2002). However, developing a self-consistent model that would incorporate the spectral and interferometric data proved to be a difficult problem. Two competing models are proposed. In one the data are explained by a disk that has a large vertical expansion (puffing) at the inner dust sublimation edge due to the direct stellar heating of the disk interior (Dullemond et al. 2001). In the other model the inner disk is surrounded by an
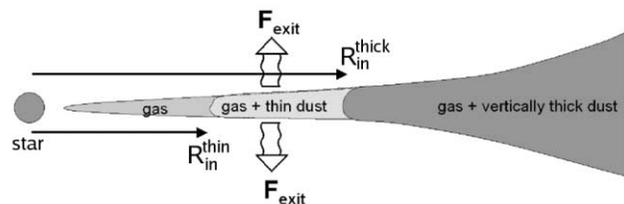

FIG. 10.—Dust dynamics in protoplanetary disks brings dust closer to the star than the inner edge of optically thick dusty disk. According to the radiative transfer solution in § 4, such a disk structure is possible if this zone is populated only by big grains and it is vertically optically thin. These conditions enable disk thermal cooling and dust survival (see also Fig. 8). The resulting sublimation zone spans from the optically thin to the optically thick inner disk radius (eq. [50]). Its physical size is ~0.2 AU in Herbig Ae stars and ~0.03 AU in T Tauri stars. [*See the electronic edition of the Journal for a color version of this figure.*]

optically thin dusty outflow, without the need for special distortions to the vertical disk structure (Vinković et al. 2006).

Although current observations cannot distinguish between these two models, theory gives some limits to the dust properties in the former model. The inner disk edge has to be populated by big dust grains ($\gtrsim 1\ \mu m$) (Isella & Natta 2005; Vinković et al. 2006) in order to produce the large and bright vertical disk puffing needed to fit the data. The hallmark of this radiative transfer solution is the dust temperature inversion, which we also demonstrate in § 2. Vinković et al. (2006) argue that purely big grains are unrealistic because dust always comes as a mixture of grain sizes (especially in dusty disks where dust collisions should constantly keep small grains in the mix). On the other hand, Isella & Natta (2005) point out that smaller grains should be sublimated away from the very surface of the inner disk edge, but cannot prove that this process is efficient enough to preserve dust temperature inversion.

Our analytical analysis in § 3 proves that multigrain radiative transfer solutions keep small grains away from the immediate surface of optically thick disks and preserve temperature inversion. This is achieved under the boundary condition of maximum possible temperature reached by all dust grains. Then the surface becomes too hot for small grains to survive, which leaves it populated only by big grains. By choosing this favorable boundary condition, we made an important presumption about the dust dynamics. Dust has to be dynamically transported to the distances where all grains start to sublimate. This can naturally occur in optically thick disks due to dust and gas accretion. However, in § 4 we discovered that big grains can survive closer to the star than the inner edge of optically thick disk. The only requirement is that the disk becomes vertically optically thin at these close distances.

Therefore, the dust sublimation zone is not a simple sharp steplike transition, but rather a large zone (Fig. 10). We estimate its size by considering the inner radius of optically thick and thin disks (Vinković et al. 2006),

$$R_{in} = \frac{\Psi R_*}{2} \left(\frac{T_*}{T_{sub}}\right)^2 = 0.0344\Psi \left(\frac{1500\ \mathrm{K}}{T_{sub}}\right)^2 \sqrt{\frac{L_*}{L_\odot}} \quad [\mathrm{AU}], \quad (50)$$

where T_*, R_*, and L_* are the stellar temperature, radius, and luminosity, respectively, and Ψ is the correction for diffuse heating from the disk-edge interior. Optically thick disks have $\Psi = 2$, while optically thin disks have $\Psi \sim 1.2$. The sublimation zone exists between these two extremes, which translates to ~0.2 AU

for Herbig Ae stars ($\sim50\,L_\odot$) or ~0.03 AU in T Tau stars ($\sim L_\odot$). This is a considerable size, detectable by interferometric imaging, and has to be addressed in future studies. Also, its gas is enriched by metals coming from the sublimated dust, which makes this zone a perfect place for dust growth. Note that, in addition to sublimation, grain growth is another way of making the disk optically thinner. Hence, the evolutionary role of such a large sublimation zone has to be studied further and in more detail.

Another major problem with the dynamics of big grains is that they tend to settle toward the midplane, which suppresses vertical expansion of the inner disk edge and leads to the failure of the disk puffing model (Dullemond & Dominik 2004; Vinković et al. 2006). On the other hand, the concurrent model incorporating a dusty outflow lacks a convincing physical process responsible for the formation of a dusty wind. The sublimation zone may play an important role in dusty wind processes, since its small optical depth and small distance from the star should also result in gas properties that are more susceptible to nongravitational forces capable of launching a dusty wind (such as magnetic fields) than the rest of the dusty disk.

6. CONCLUSION

We analyzed the temperature structure of externally heated optically thick dust clouds. We focused on the recently discovered effect of temperature inversion within the optically thin surface of a cloud populated by big ($\gtrsim1\,\mu$m) dust grains (Isella & Natta 2005; Vinković et al. 2006). The effect is produced by reprocessing the external radiation and not by any additional energy source. The inversion manifests itself as a temperature increase with optical depth, before it starts to decrease once the external directional radiation is completely transformed into the diffuse thermal flux. Since small grains do not show this effect, the open question was whether small grains would manage to suppress this effect when mixed with big grains.

We show analytically that small grains remove the temperature inversion of big grains if the overall opacity is dominated by small grains. However, this does not happen in situations where the cloud is close enough to the external energy source for dust to start sublimating. Small grains acquire a higher temperature than bigger grains and sublimate away from the immediate cloud surface. The exact grain size composition of the surface depends on the amount of external flux because different grain sizes sublimate at different distances from the surface. These distances are smaller than in optically thin clouds because bigger grains shield smaller grains from direct external radiation. We show that the temperature inversion is always preserved if small grains are removed from only $\tau_V \sim 1$ of the immediate surface.

If the boundary condition requires all dust sizes to maximize their temperature, then all small grains are removed from the surface layer. They can exist only within the cloud interior $e^{-\tau_V} \ll 1$, where the external radiation is completely absorbed. Such a condition is expected in protoplanetary disks, where dust accretion moves dust toward the star. The inner disk radius is then defined by the largest grains, no matter what the overall grain size composition, because the largest grains survive the closest to the star and dictate the surface radiative transfer.

A new problem arises in that case. Since the temperature inversion keeps the very surface of the cloud below the sublimation temperature, its dust can move even closer to the star. We show that this creates an optically thin dusty zone inside the dust destruction radius of an optically thick disk (Fig. 10). Only big grains can survive in this zone. We estimate that its size is large enough to be detected by near-IR interferometry. It consists of big grains and gas enriched by metals from sublimated dust, hence favorable for grain growth. This shows that the geometry and structure of inner disks cannot be determined by simple ad hoc boundary conditions. It requires self-consistent calculations of dust dynamics combined with radiative transfer calculations and dust sublimation.

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