Schelling's model with income preferences and a housing market

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1 Model description

The final form of the model adopted in our study consists of three utility components describing racial preference (U_R), income preference (U_I) and price of housing (U_P). The racial preference is defined as the measure of the affinity of an agent to move into a location that contains some number of neighbors (within the Moore neighborhood) of the opposite race. Although its functional form can be anything, we use the original step function description by Schelling

$$U_R = \begin{cases} u_R & \text{if } \rho \leq \rho_R \\ 0 & \text{if } \rho > \rho_R, \end{cases} \tag{1}$$

where $\rho \in [0,1]$ is the proportion of unlike neighbors, while $u_R \geq 0$ and $\rho_R \in [0,1]$ are given model parameters. As we will see later from numerical results, this simple definition is sufficient to yield a plethora of interesting results and model outcomes. The income preference is similar, except that here everybody wants to live in a rich neighborhood. Hence, the utility depends exclusively on the fraction of poor neighbors, $\gamma \in [0,1]$, irrespective of who makes the move:

$$U_I = \begin{cases} u_I & \text{if } \gamma \leq \gamma_I \\ 0 & \text{if } \gamma > \gamma_I. \end{cases} \tag{2}$$

The variables $u_I \geq 0$ and $\gamma_I \in [0,1]$ are free model parameters. Finally, the utility describing satisfaction of an agent at the location (x,y) and moment t with the price of housing $u_P P(x,y,t)$ is

$$U_P(x,y,t) = -\sigma_I \sigma_R u_P P(x,y,t), \tag{3}$$

where $u_P \geq 0$ and σ_I and σ_R are defined below. The price is changing in time according to changes in demand $D(x,y,t)$ for this location. The unit of time is one iteration step equivalent to moving one agent. Prices are updated after one "*price cycle*" of time length t_c

$$P(x,y,t) = \begin{cases} D(x,y,t)/N_{tot} & \text{if } t/t_c \text{ is an integer} \\ P(x,y,t-1) & \text{if } t/t_c \text{ is not an integer .} \end{cases} \tag{4}$$

Demand $D(x,y,t)$ is determined by counting all agents in the model that would prefer moving to (x,y). The theoretical minimum demand is zero and the maximum is equal to the total number of people N_{tot} in the model. Since the rich do not consider given prices to be of the same subjective value to them as for the poor, we introduce a correction factor in equation 3

$$\sigma_I = \begin{cases} \in [0,1) & \text{if rich} \\ 1 & \text{if poor.} \end{cases} \tag{5}$$

Also, studies have revealed that whites are in average richer than blacks, which is incorporated into equation 3 as

$$\sigma_R = \begin{cases} \in (0,1] & \text{if white} \\ 1 & \text{if black.} \end{cases} \tag{6}$$

One can imagine that sometimes racial preferences are stronger for those in worse economic situation (e.g., blaming blacks or immigrants for their economic problems). In that case there would be an additional race-dependent function multiplied with the housing price utility in such a way that poor living in poor neighborhoods would be more race intolerant. This can be introduced into the model, but we do not consider it because it obviously enhances racial segregation, while our goal is to explore effects of the most basic example of housing price utility.

The final total utility is defined as

$$U_{tot}(x,y,t) = U_R(x,y) + U_I(x,y) + U_P(x,y,t). \tag{7}$$

Models are parameterized with a set of free parameters: (N_{wr}, N_{wp}, N_{br}, N_{bp}, u_R, ρ_R, u_I, γ_I, u_P, σ_R, σ_I, t_c), where N_{wr} is the number of white reach people, N_{wp} of white poor people (hence, the number of whites is $N_{wr} + N_{wp} = N_w$), N_{br} of black rich people and N_{bp} of black poor people ($N_{br} + N_{bp} = N_b$ of blacks; $N_w + N_b = N_{tot}$). Numerical experiments on a computational grid of $N_x \times N_y$ in size also contain $N_{empty} = N_x N_y - N_{tot}$ number of unoccupied locations (vacancies).

A simulation sequence starts with the initial random positioning of agents and setting all prices and demands to zero ($P(x,y,0) = 0$ and $D(x,y,0) = 0$). The iteration loop consists of randomly selecting an agent that needs to increase its utility (that is, an agent with $U_{tot} < u_R + u_I$) and moving it to a randomly chosen vacancy where the agent's utility would be larger. The demand for each location, irrespective of being occupied or not, is calculated after t_c iteration steps, followed by an update of the price of housing.

2 Theoretical predictions

In our previous study (Vinković & Kirman, 2006) we described a mathematical transformation of the Schelling model into a physics model of clustering in liquids formed by surface tension force. Here we briefly outline this procedure and apply it on the utility in equation 7, while for the details we point the reader to our previous paper.

In the limit of infinitesimally small lattice cells, a population of agents at a point $\vec{r}$ is described by their number density per unit area $n(\vec{r}) = dN(\vec{r})/dA$. In this limit, the number of neighbors used in U_R and U_I is replaced with the solid angle θ around the point $\vec{r}$ covered by particles of a given type. We replace the race and income utility functions with energies $\varepsilon_R(\theta)$ and $\varepsilon_I(\theta)$, where a high utility corresponds to a low energy and vice versa. The motivation for this comes from physics where particles always tend to minimize their energy, while in economy agents want to maximize their utility. Since we do not allow more than one agent per lattice cell, n is constant and we can set it to $n = 1$ (one particle per unit area).

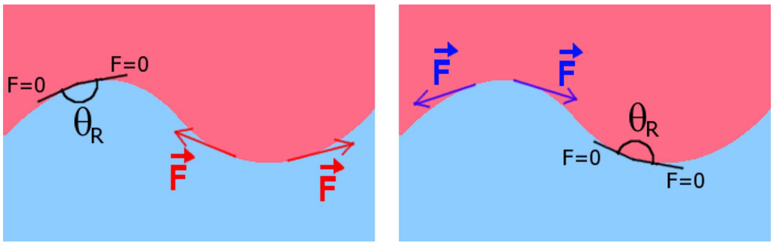

Figure 1: Growth and evolution of racial clusters is dictated by the forces on their surface. Red and blue color indicate two different races that are mathematically equivalent when only race is considered (see equation 1). The left panel shows forces on the red cluster. Forces are reducing the size of surface perturbations until the surface curvature reaches the optimal value defined by θ_R. The right panel shows the same situation for the blue cluster.

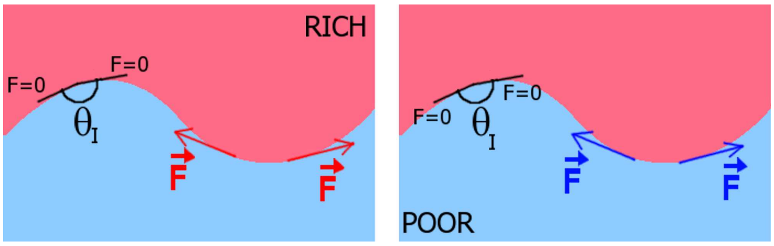

Figure 2: The rich cluster (red color) shown in the left panel behaves similarly to racial clusters (figure 1), except that the surface curvature is defined by θ_I. The poor cluster (blue color) shown in the right panel, on the other hand, is unstable to perturbations because the income preference defined in equation 2 depends exclusively on the number of poor neighbors. This leads to surface forces that expand surface perturbations and prevent the cluster from growing. Any attempt to form a poor cluster is, therefore, quickly stopped and the cluster disintegrated due to its diffusion into the rich cluster.

Evolution of a cluster can be predicted by the forces acting on its surface. The surface forces can either provide stability to the cluster by reinforcing its surface against perturbations or working on ripping the cluster apparat. In our case there are two types of clusters and accompanying surfaces that can coexist: racial clusters (blacks and whites) and income clusters (poor and rich). We can derive the surface tension force from its definition as a gradient of energy along the surface. Gradient of energies based on utilities in equations 1 and 2 is a step function: the forces are non-zero and equal to $|\vec{F}_R| = u_R$ and $|\vec{F}_I| = u_I$ only on cluster surfaces with curvatures *larger* than $\theta_R = 2\pi\rho_R$ and $\theta_I = 2\pi\gamma_I$. This means that ρ_R and γ_I predispose the cluster evolution. Formally written, the energy at a surface point $\vec{r}$ along an infinitesimal $d\vec{L}$ following the surface is $n\varepsilon_R(\theta)dL$ and $n\varepsilon_I(\theta)dL$, while the forces are $\vec{F}_R(\vec{r}) = -n\hat{L}\varepsilon_R(\theta)$ and $\vec{F}_I(\vec{r}) = -n\hat{L}\varepsilon_I(\theta)$, where $\hat{L}$ is the unit vector tangential to the cluster surface.

The growth and evolution of racial clusters has been already described by Vinković & Kirman 2006. Notice in their figures 3 and 4 that clusters grow even in cases when racial tolerance is slightly over 50% of unlike neighbors ($\rho_R \leq 5/8$). In figure 1 we illustrate the influence of surface forces on the cluster evolution.

The income utility is slightly different because here the opposite clusters behave differently due to the rule that all agents like to move into rich neighborhoods. Therefore, forces on rich clusters try to evolve them as they do in the case of racial clusters, but at the same time poor clusters behave in exactly the opposite way. The surface of poor clusters in unstable for perturbations; any surface bump is going to grow and push into the rich cluster, destroying the homogeneity of both clusters. In other words, poor clusters quickly evaporate and deposit their particles into the rich cluster. This leads to an equilibrium of perfectly mixed poor and rich agents (ref?), unless some other force (that is, an energy or utility "barrier") is introduced to prevent the poor diffusing into the rich clusters. Figure 2 visualizes these forces on poor and rich clusters.

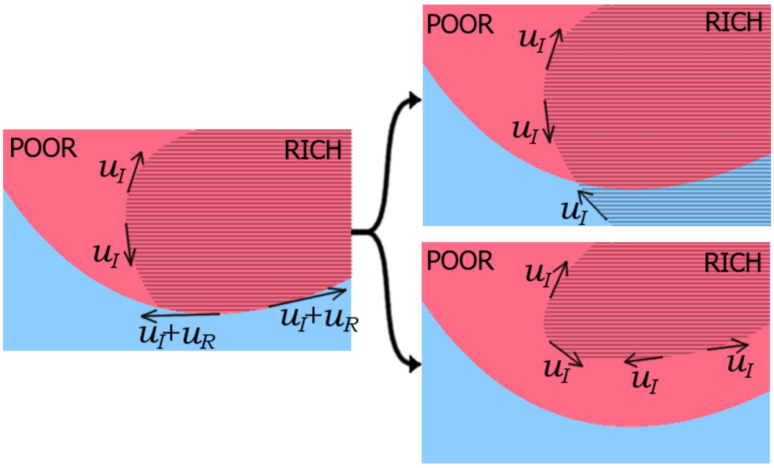

Figure 3: Racial and income clusters evolve independently of each other except in the case when they share a surface. The left panel shows racial clusters in color and a rich cluster as a shaded area. The part of rich surface shared with the racial surface imposes the largest forces on the rich cluster. This surface will, therefore, evolve faster than the rest of the surface into one of two configurations shown in the right panels.

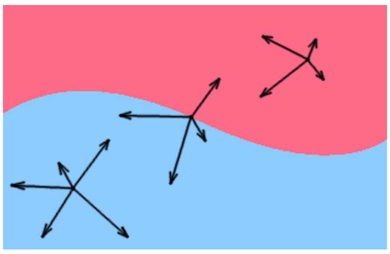

Figure 4: Unlike racial and income forces illustrated in figures 1-3, the force due to price of housing behaves like a pressure present at all points in space. The strength of this force depends on the local gradient of the price of housing and can differ for different directions at the same given point.

According to equation 7, racial and income utilities are additive components of the total utility. This means that racial and income components of the system evolve independently according to their own separate forces. Hence, racial segregation will proceed unaffected by the income preference unless it is a negligible component to the total utility ($u_R \ll u_I$). The only instance where these two utilities interact is when a racial cluster and an income cluster share a surface. Assuming that under a certain condition a rich cluster has formed and it shares a part of its surface with a racial cluster, then its surface force is everywhere either zero or $|\vec{F}_I| = u_I$, except on the shared surface where it is $|\vec{F}_I+\vec{F}_R| = u_I + u_R$. This net force on the shared surface is, therefore, the largest surface tension force and it deforms the surface faster than the rest of the rich cluster. There are two ways how to reduce this force: either extend the rich cluster over the racial barrier or contract it away from the barrier (see figure 3). This yields an important conclusion: *extremes avoid each other*, that is, rich-white cluster will avoid sharing a part of it surface with poor-black. The same for rich-black and poor-white.

The force due to the housing price utility behaves differently than racial and income forces. The utility U_P depends only on its local value and it is equivalent to a local potential energy $\varepsilon_P(\vec{r}, t)$.

The housing price force is a gradient of this scalar field $\vec{F}_P(\vec{r},t) = -\vec{\nabla}\varepsilon_P(\vec{r},t)$ and it behaves like a pressure trying to either expand or compress the clusters. This pressure is forcing particles to follow the price gradient and flow toward regions of lower prices. Unlike racial and income forces, which are zero unless the direction is tangential to the cluster surface, the price pressure force acts in all directions, although with different amplitudes (see figure 4). Nevertheless, additive property of utilities in equation 7 still implies that *racial segregation will appear whenever the racial utility provides a non-negligible contribution to the total utility*. Notice that equation 3 yields variations in segregation depending on the race and income. Segregation will not occur in poor whites if $u_R \ll u_P\sigma_R$, rich whites if $u_R \ll u_P\sigma_I\sigma_R$, poor blacks if $u_R \ll u_P$ and rich blacks if $u_R \ll u_P\sigma_I$. As already described above, it will not occur in all groups if $u_R \ll u_I$.

3 Numerical results

3.1 The baseline model

There is obviously a plethora of possibilities to explore with such a large set of free parameters introduced by the model in §1. Instead of exhausting all possibilities, we decided to focus on one set of parameters that captures some basic observed characteristics of racial segregation in the United States and then explore what happens when we individually vary these parameters. We call this basic parameter set the "baseline model" and it consists of the following:

$$N_{wr} = 2250,\, N_{wp} = 2250,\, N_{br} = 2250,\, N_{bp} = 2250$$
$$u_R = 1,\, \rho_R = 0.5,\, u_I = 1,\, \gamma_I = 0.5,\, u_P = 5,\, \sigma_R = 0.7,\, \sigma_I = 0.1,\, t_c = N_{empty}$$

Models are calculated on 100×100 non-periodic grid, thus, the number of empty space is $N_{empty} = 1000$. Price update cycle can be either short (comparable to N_{empty}) or long (comparable to N_{tot}). During a short cycle each vacant location becomes occupied once in average, while during a long cycle each agent makes at least one move in average. We choose a short cycle for the baseline model because it is more close to a realistic price dynamics in cities, although we also explore what happens when the cycle is long.

The numerical experiments start with the initial price of zero everywhere. The system evolves quickly within the first ∼20,000 iterations when the totaly system utility becomes very close to its equilibrium value. After that the total utility (averaged over a price cycle) grows very slowly, while the system is evolving further by rearranging particles in order to optimize the clusters' size and shape. This change in the rate of system evolution happens because initially agents form the smallest clusters possible that would give them the largest boost to their utilities. After that changes in demand are relatively small, which results in small changes of utility for the majority of agents. The evolution becomes dominated by changes in the clusters' shape and size, with only a small number of agents experiencing big utility change during a price cycle by gaining on racial or income utility. But this slow evolution due to constantly varying scalar field of housing prices is an important new feature in the Schelling model. Without this time depended field the system would quickly approach some equilibrium and freeze. Therefore, in our previous study (Vinković & Kirman 2006) we had to introduce an additional rule of letting agents move when their utility remains constant. This produced a "liquid" behavior of the system. Here we do not need this rule because *small variations in individual utilities due to changes in the housing price drive the system to behave like a liquid.*

In the figures that follow we show the model properties after 1,000,000 iterations, unless it is indicated otherwise. In addition to two-dimensional spatial distribution of agents, we also follow two-dimensional spatial distribution of housing prices, distribution of agents of different type over the full range of housing prices (from 0 to $u_P = 5$), distribution of agents of different type over a range of individual utility values (from -3 to the maximum of $u_R + u_I = 2$) and the measure of segregation in income and race for different sizes of "city blocks". Distributions over housing prices and individual utilities are calculated by dividing the prices and utilities into 30 bins. Statistical fluctuations are somewhat reduced by averaging distributions over the last 10% of iterations within a price cycle. The measure of segregation for a city block of $\kappa \times \kappa$ in size is calculated as a sum of deviations from the overall average fraction of blacks or whites:

$$S(\kappa) = \frac{1}{N_\kappa}\sum_{i=1}^{N_\kappa}\left|\frac{N_b}{N_{tot}} - \frac{N_{b,i}}{N_{b,i}+N_{w,i}}\right| = \frac{1}{N_\kappa}\sum_{i=1}^{N_\kappa}\left|\frac{N_w}{N_{tot}} - \frac{N_{w,i}}{N_{b,i}+N_{w,i}}\right|, \tag{8}$$

where $N_\kappa = (N_x - \kappa)(N_y - \kappa)$ is the total number of possible positions of the city block within the city and $N_{b,i}$ and $N_{w,i}$ are the number of blacks and whites within a city block i. The same function can be written for income segregation if indexes for black and white are replaced with poor and rich. Initial random positioning of agents introduces some unindented segregation. This constitutes the minimum segregation and it is marked by a dotted line in segregation plots. Double-counting prevents an easy estimate of the maximum segregation value, but what is important to observe is how strongly it deviates from the minimum value due to random distribution. Notice that the level of segregation differs for different sizes of city blocks. In general, segregation is stronger on smaller scale, while the city block equal to the whole city in size has zero segregation by definition.

The result of baseline model is shown in figure 5. Racial segregation reached the level of "complete" segregation where racial clusters will eventually merge into one big cluster (for details see Vinković & Kirman 2006). Spatial segregation by income is almost negligible. It exists only on small scale and it is not capable of evolving into larger clusters. This is also visible on the spatial distribution of prices, which does not indicate any large price cluster forming. However, the number of people in various price groups shows that richer agents occupy more expensive housing. They can afford this because their subjective view of these prices is much smaller ($\sigma_I = 0.1$) than for poor people. If only the poor side of the income distribution is considered (housing prices below ~ 2), the whites in average occupy slightly higher housing prices as expected from $\sigma_R = 0.7$. Notice how different segregation properties coexist in this system: spatial racial segregation, but no spatial income segregation, while income segregation in housing price distribution. This is important to

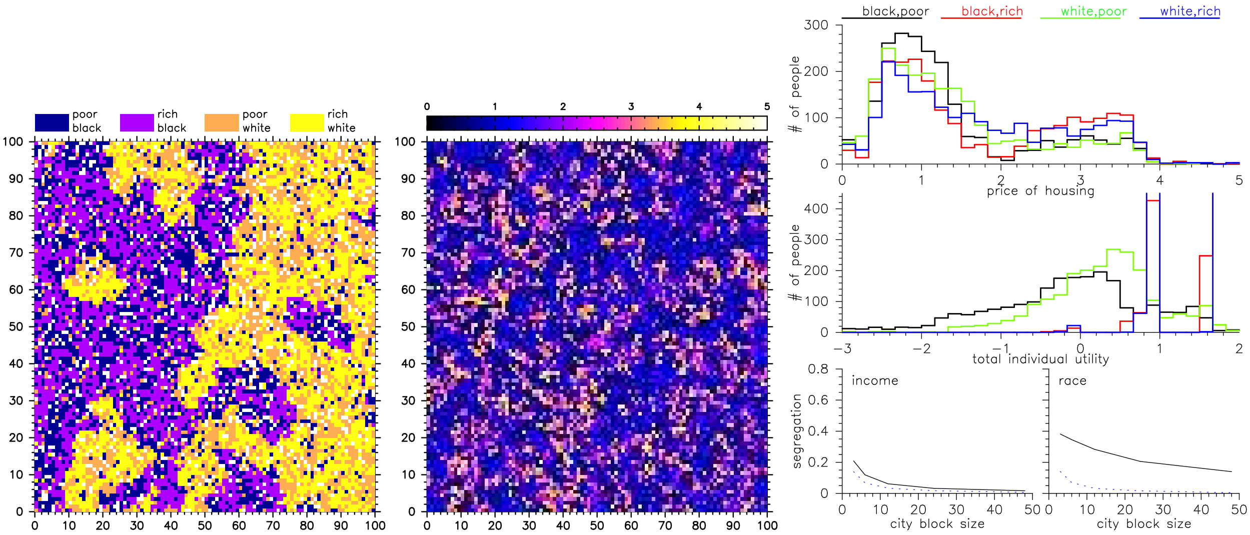

Figure 5: The baseline model (for details see §3.1) shows spatial racial segregation, but no significant spatial income segregation. The far left panel is a spatial distribution of agents of different types. The middle panel is a spatial distribution of housing prices. The lack of noticeable clustering of prices is an indication of a negligible spatial income segregation. The far right panels are distributions over housing prices, individual utility values and measures of segregation (see equation 8; dotted line is segregation due to random positioning of agents). The system segregates in income over housing prices, even though it does not show spatial income segregation.

understand because any partial study that would look only at one property (for example, distribution over housing prices) can yield misleading conclusions.

3.2 Reducing the amount of empty space

The empty cells play a very important role in the Schelling model. In Vinković & Kirman (2006) we explained how empty space takes the role of a boundary layer that can stabilize a cluster surface that would undergo deformations if in direct contact with the unlike cluster. This is why wide streets/avenues or city parks often become boundaries of racial and income clusters in urban areas. When we reduced the number of empty cells in our baseline model from 1000 to 400, an extensive spatial income segregation emerged (see figure 6). Poor clusters are also identified with uniformly low housing prices, while rich clusters experience dramatic price changes from very hight to very low. This price oscillations happen because in one price cycle the price is so high that only rich people can afford it, which drastically reduced the demand for these locations and deflates their prices to very low in the next cycle. Very low prices mean a high demand and the prices are inflated again to high values in the next cycle, which perpetuates the price oscillations.

It is easy to see why income clusters emerge under these conditions. If a poor agent manages to find a very low price vacant spot in a rich cluster it will occupy it. However, in the next price cycle the housing price of this location is going to increase above what the poor agent can afford

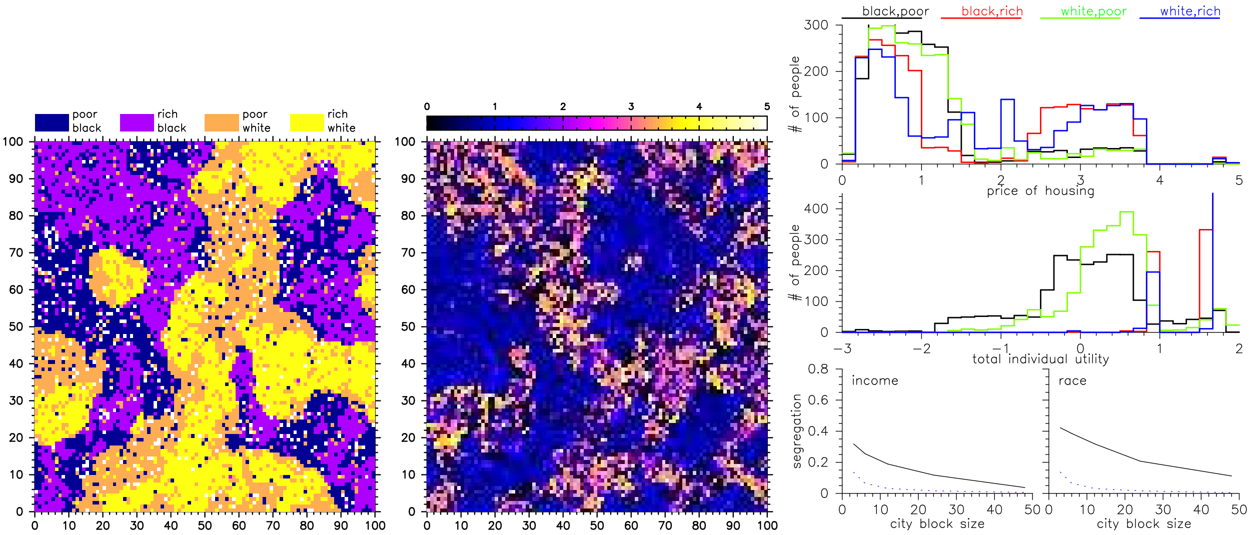

Figure 6: The model with reduced number of empty cells (400 in this example: $N_{wr} = N_{wp} = N_{br} = N_{bp} = 2400$) shows an additional spatial income segregation in comparison with the baseline model (figure 5). This is also visible in the middle panel where poor clusters have low housing prices, while rich clusters experience very dynamic price changes from high to low. Notice also that the highest prices largely follow boundaries of racial clusters and how extremes try to avoid each other (rich white clusters avoid poor black, and rich black avoid poor white).

and it will be forced to move out. On the other hand, a rich agent is far more resilient to price changes because housing prices do not affect its utility as much. Reduced number of empty cells in the system also reduced the probability for poor agents to find a low price housing in rich clusters. This resulted in increased stability of rich clusters and gave them ability to grow. As we already predicted in §2, emergence of rich clusters also leads to the phenomenon of extremes avoiding each other. Rich white clusters tend to be separated from poor black clusters by a zone of either poor whites or rich blacks. Similarly, rich blacks avoid poor whites. An additional feature is noticeable in the case of simultaneous emergence of rich and income clusters: the highest prices follow racial clusters' boundary within rich clusters. This is due to an extra demand for these locations: not only that all the poor would like to be in these rich locations, but also a big fraction of black *and* white rich agents would benefit in their racial utility from moving to this boundary.

3.3 Slowing down the price cycle

If housing prices do not change then agents move until they maximize their utility relative to the all available empty cells and the system freezes. Housing price cycles long enough to be comparable with this "freezing time" are govern by the minimal demand possible since in each cycle agents have enough time to maximize their utilities. The net result is price distribution for poor agents shifted toward very low prices where the housing price utility cannot compete with the racial and income utilities. The system segregates into rich and poor clusters, with rich clusters experiencing large

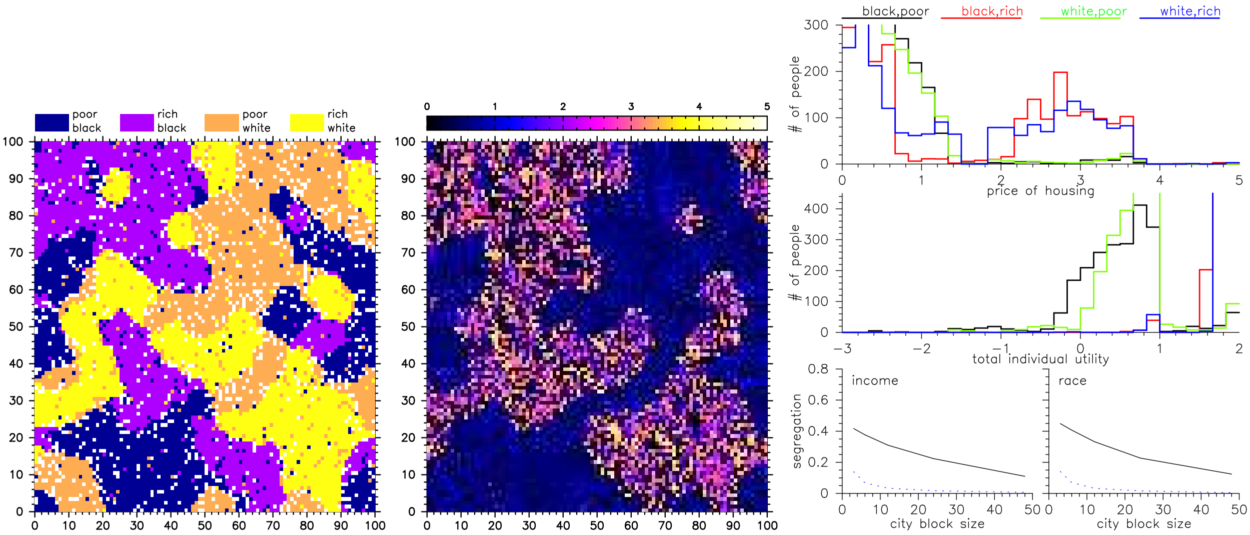

Figure 7: Same as the baseline model, but for $t_c = N_{tot} = 9000$. Results are similar to figure 6.

price oscillations, which enhances income segregation as already described in §3.2. Figure 7 shows the outcome of this model when the price cycle is $t_c = N_{tot}$. It shares similarites with the model in §3.2, including the tendency of the highest prices to follow racial borders and tendency of extremes to avoid each other (separated at least by a monolayer of intermediate agent types or empty cells - see discussion in §3.2).

3.4 Small racial utility

If the racial utility is completely removed from the model, the system becomes completely racially integrated. We explored the emergence of racial segregation by increasing the amplitude of racial utility from zero to its baseline value of one. Figure 8 shows results for $u_R = 0.1$ and $u_R = 0.5$. Racial segregation becomes important at $u_R \sim 0.5$, while reducing the number of empty space at $u_R = 0.5$ increased racial clustering within the rich population. The case of $u_R = 0.5$ is interesting because it equals the maximum housing price for rich black ($u_P \sigma_I = 0.5$). A certain level of clustering emerges, but it is not capable of forming large clusters. Also, black clusters that emerge are dominantly rich blacks and housing price distribution starts to differ between the rich whites and rich blacks. Reducing the number of vacant locations amplifies racial segregation in the $u_R = 0.5$ case, but only for the rich. This means that income segregation is also increased. The poor resist significant clustering because the racial utility is not strong enough to compete with their housing price utility.

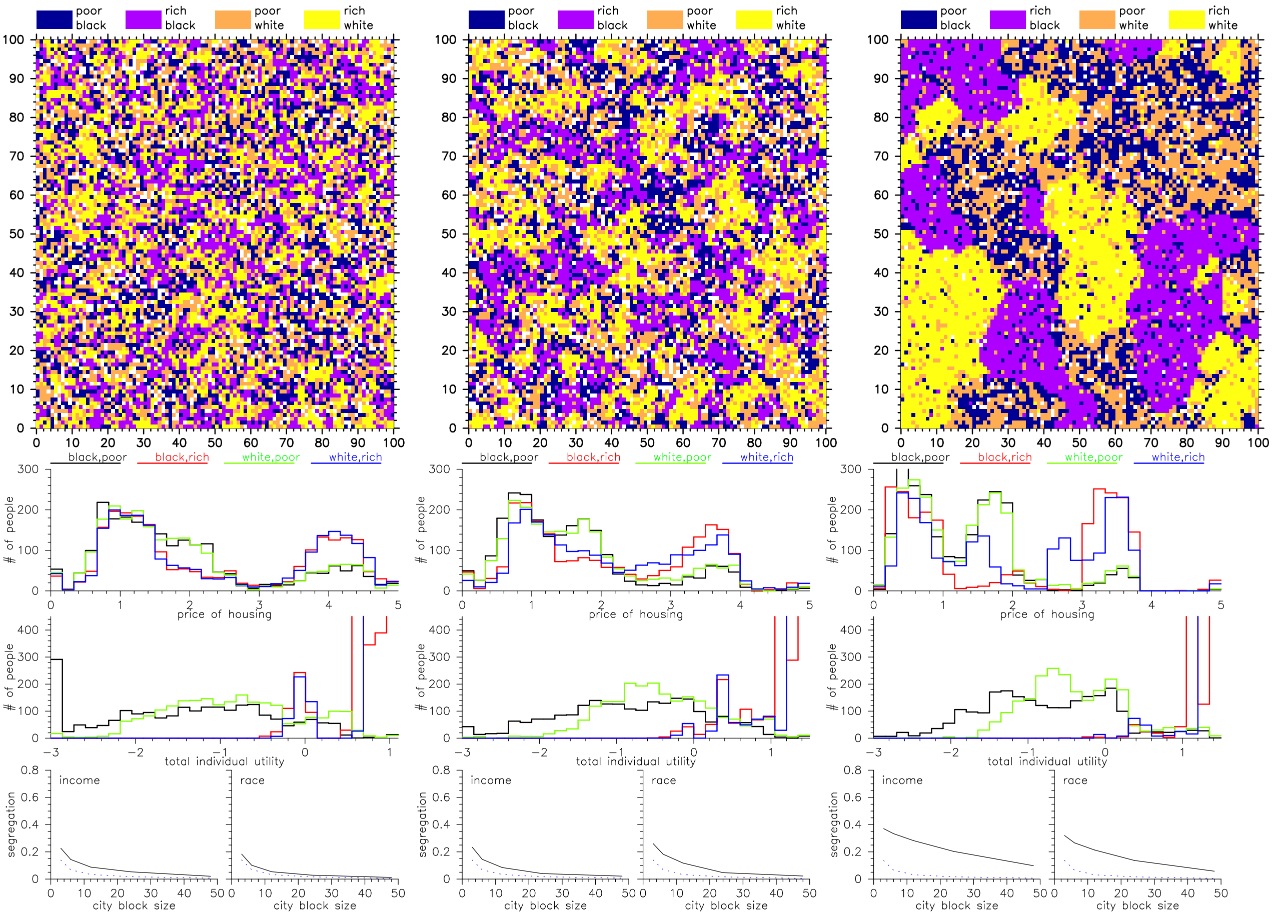

Figure 8: Same as the baseline model, but for $u_R < 1$. The far left column shows results for $u_R = 0.1$, the middle column for $u_R = 0.5$ and the far right column for $u_R = 0.5$ *and* $N_{empy} = 400$ ($N_{wr} = N_{wp} = N_{br} = N_{bp} = 2400$).

3.5 A poor city

Segregation disappears when the fraction of poor people in the system is increased above ∼70%. Housing price distribution indicates two major price groups: one occupying low housing prices that compete with the racial utility and the other with high housing prices that dominate over the racial utility. Since there is no enough rich people to fill in the expensive locations, poor agents at high price locations are constantly forced to move. As soon as they move to a new place, a new pattern of high prices emerge and racial clusters cannot consolidate. Reducing the number of empty cells does not help because this does not provide an antidote for the lack of rich agents that could occupy expensive locations.

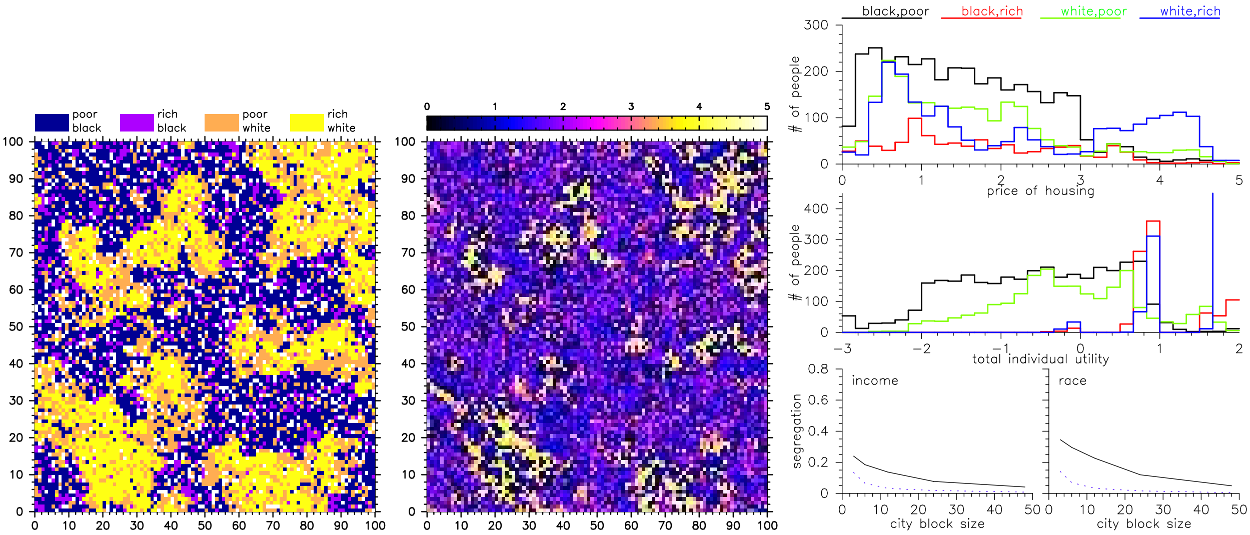

Figure 9: Same as the baseline model, but for blacks being mostly poor ($N_{bp} = 3600$, $N_{br} = 900$). Racial segregation under these conditions also leads to income segregation with white clusters dominated by the rich. Notice how different apparent behaviors of rich whites (who cluster) and rich blacks (who do not cluster) is just a side-effect of the model and not an intrinsical behavioral property of agents.

3.6 Mostly poor blacks

An important aspect of the racial segregation issue is the income class distribution of black households. The baseline model is designed to reflect the current situation where approximately one-half of black Americans live in middle- or upper-income households. But in 1960 this fraction was about one-fifth (Council of Economic Advisors 1998; U.S. Census Bureau 2000). We looked at the behavior of our model when the fraction of rich blacks is reduced to 20% of the black population and noticed an interesting phenomenon (see figure 9). Racial clustering in this model leads to income segregation in whites, with an additional property of white clusters being mostly rich. This is a direct consequence of black clusters being also poor clusters by their nature. Since rich white clusters avoid a direct contact with poor black clusters, they are forced to move into the interior of white clusters, while poor whites occupy the boundary between white and black clusters. In addition, a large fraction of poor whites scatters into the poor black neighborhoods in search for lower housing prices, which leaves white clusters richer in average. Rich blacks, on the other hand, do not cluster because of their small number (any attempt to cluster is quickly disturbed by a large influx of poor blacks). Hence, *different apparent behavior of rich whites and rich blacks is not the cause of segregation in this model, but a consequence of it.*

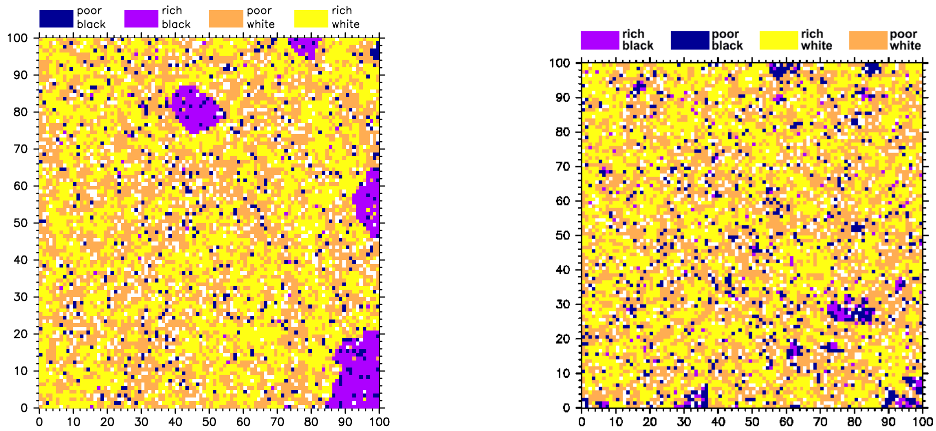

Figure 10: Same as the baseline model, but for a small number of blacks ($N_w = 8000$, $N_b = 1000$). The left panel shows the model with $N_{br} = N_{bp} = 500$, while the right panel has mostly poor blacks ($N_{br} = 200$ and $N_{bp} = 800$). Blacks manage to cluster only in the case of rich blacks when they constitute a big fraction of the black population (left panel).

3.7 Black minority

Farley and Frey (1994, p. 40) observe that "the largest decreases in segregation occurred in metropolitan areas in which blacks made up a small percentage of the neighborhood of the typical white." We explore this by reducing the number of blacks to 1000 in figure 10 and find that our model is consistent with this observed trend. Only rich blacks are capable of clustering when they constitute a big fraction of the black population. If they are a minority then a stable black cluster does not emerge. The poor black majority is moving into emerging rich black clusters and destroy them at the same time.

3.8 Asymmetric racial preferences

Cutler et al. (1999) showed theoretically that relative differences between housing prices of blacks and whites can indicate which race is exercising larger racial preference and causing segregation. If segregation is caused by white preferences for white neighborhoods, then whites will pay relatively more for housing than blacks as segregation rises. This will reduce the relative housing costs of blacks compared to those of whites. If discrimination and/or black preferences for black neighborhoods are the causes of segregation, then blacks will pay relatively more for housing than whites in more segregated cities. This will increase housing costs for blacks relative to whites.

We introduced asymmetric racial preferences between blacks and whites in order to test the ability of our model to reproduce these trends. In one model blacks were more tolerant than whites ($\rho_R = 5/8$ for blacks and $\rho_R = 3/8$ for whites), while in the other model the opposite was the case.

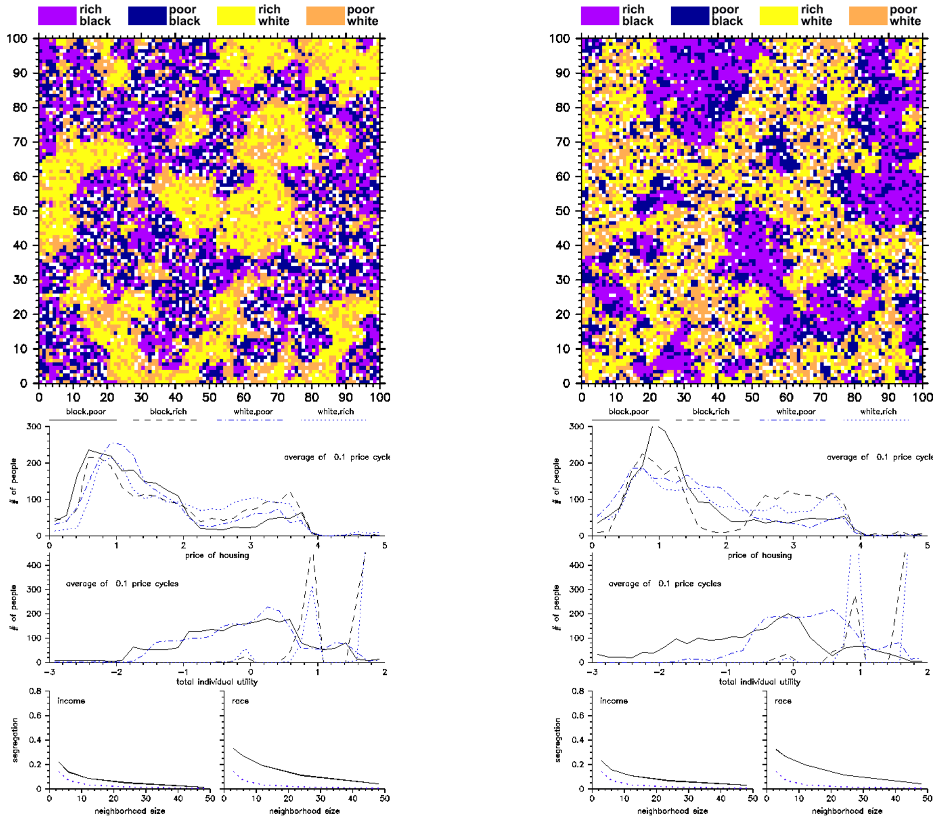

Figure 11: Same as the baseline model, but for different racial preferences of blacks and whites. The left column is the model with blacks ($\rho_R = 5/8$) being more tolerant than whites ($\rho_R = 3/8$). The right is the opposite situation, where whites ($\rho_R = 5/8$) are more tolerant than blacks ($\rho_R = 3/8$). In both cases the less tolerant racial group creates racial clusters with dominantly rich population that can afford to live in this high demand racial clusters. Notice how the housing price distribution for blacks shifts toward lower prices in average when blacks are more tolerant (hence, increasing the price gap between blacks and whites) and toward higher prices in average when they are less tolerant (hence, decreasing the housing price gap).

Figure 11 shows the results. Distribution of housing prices shows the trend predicted by Cutler et al. (1999) (compare these trends also with the baseline model in figure 5). But also an additional pattern emerged: the less tolerant racial group creates mostly rich racial clusters. This is a direct consequence of increased demand for race intolerant clusters, while the demand for race tolerant clusters decreased (that is, the need for clustering is low in the case of race tolerant agents). The rich can afford the inflated housing price of race intolerant clusters, while the race intolerant poor have a difficulty clustering because as soon as they start forming a cluster the housing price increases.

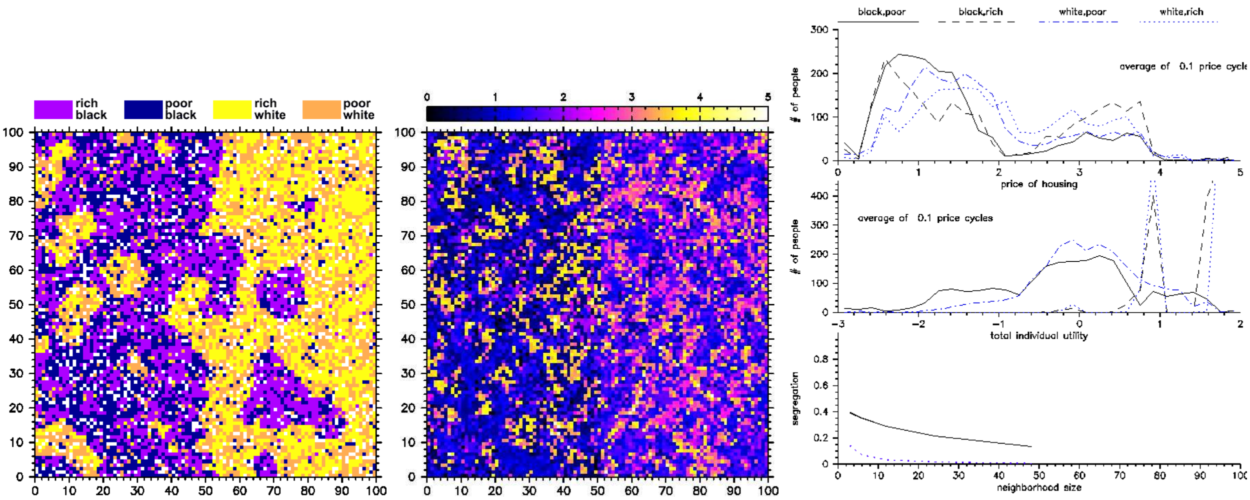

Figure 12: Same as the baseline model, but with an additional initial housing price of 0.5 for locations at $x > 50$. Whites occupy the side with higher prices because they are richer in average ($\sigma_R = 0.7$). This shows 300,000 iterations.

3.9 Predefined housing qualities

In reality, housing prices are not only a function of demand but also of their infrastructure quality or attractive location. This constitutes an additional non-uniform background field of housing prices $P_{stat}(x, y)$. We explored the most simple example where

$$P_{stat}(x, y) = \begin{cases} 0 & \text{if } x \leq 50 \\ 0.5 & \text{if } x > 50. \end{cases} \tag{9}$$

According to Sethi and Somanathan (2004), this should lead to racial segregation with whites occupying the more expensive half of the city because they are richer in average and can afford it. Indeed, this is what our model shows (figure 12). If the static background price for $x > 50$ is somewhat larger than $\sim$1 then the main separation between the two sides is due to rich and poor, with racial segregation continuing within each side. If the initial non-uniform price is very small, it does not affect the model outcome.

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